STATISTICAL CHARACTERIZATON OF SOFT TISSUE INELASTIC MATERIAL PARAMETERS AND APPLICATION TO THE MATERIAL FAILURE.

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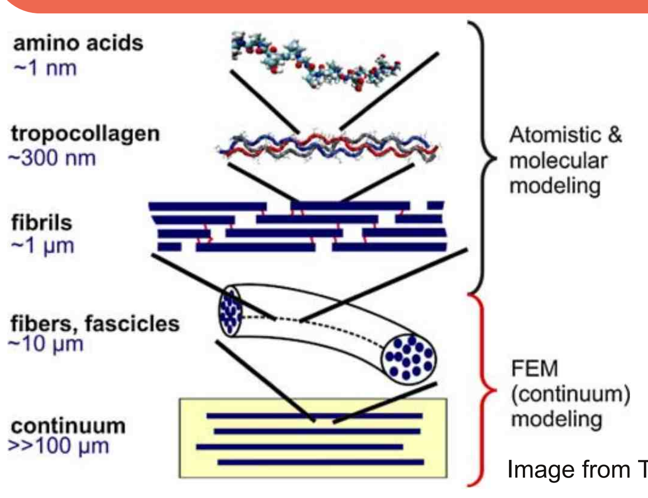
1. INTRODUCTION

1. Hierarchical structure of soft tissues



Soft collagenous tissue features many hierarchies of structure, starting from tropocollagen molecules that form fibrils, and proceeding to a bundle of fibrils that form fibers.

Image from Tang, H., Buehler, M. J., & Moran, B. (2009)

2. Summary of the work

- We present a continuous damage model with regularized softening (smeared crack models) for fiber reinforced soft tissues.
- Material parameters of the continuous model derive from the mesoscopic scale.
- In the mesoscopic scale continuum is considered as a collagenous fibril-reinforced composite.
- We want to study the continnum-level response as a function of the nanoscale properties of the collagen and the adherent forces between the tropocollagen molecules.

2. METHODS

2.1 Regularized damage model

1. Strain energy function

Degradation is modeled independently in the matrix and in each family of fibers

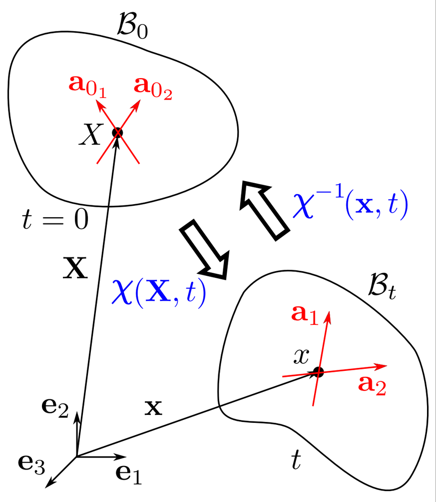
$$W = \underbrace{\frac{1}{2}K(J-1)^2}_{U(J)} + (1-d_g)\underbrace{\frac{1}{2}c\left(\bar{I}_1-3\right)}_{\overline{W}_g} + (1-d_{f1})\underbrace{\frac{k_{11}}{2k_{21}}\left(e^{k_{21}\bar{E}_1^2}-1\right)}_{\overline{W}_{f1}}$$

$$+(1-d_{f2})\underbrace{\frac{k_{12}}{2k_{22}}\left(e^{k_{22}\bar{E}_2^2}-1\right)}_{\overline{W}_{f2}}$$

$$\bar{I}_1 = \mathrm{tr}\left(\overline{\boldsymbol{C}}\right);\ \bar{I}_2 = \frac{1}{2}\left[\mathrm{tr}\left(\overline{\boldsymbol{C}}\right)^2 - \mathrm{tr}\left(\overline{\boldsymbol{C}}^2\right)\right];\ \bar{I}_3 = \det\overline{\boldsymbol{C}} = 1$$

$$\bar{I}_4 = \overline{\boldsymbol{C}} : (\boldsymbol{a}_{0_1}\otimes\boldsymbol{a}_{0_1});\quad \bar{I}_6 = \overline{\boldsymbol{C}} : (\boldsymbol{a}_{0_2}\otimes\boldsymbol{a}_{0_2})$$

$$\bar{E}_1 = \overline{\boldsymbol{C}} : \boldsymbol{H}_1 - 1;\quad \bar{E}_2 = \overline{\boldsymbol{C}} : \boldsymbol{H}_2 - 1$$



2. Softening regularization

Model dissipation

$$\mathcal{D}_{\mathrm{INT}} = \sum_{\alpha=g,f1,f2}\dot{d}_\alpha\overline{W}_\alpha = \sum_{\alpha=g,f1,f2}\frac{1}{2}\left(q_\alpha\dot{r}_\alpha - \dot{q}_\alpha r_\alpha\right)$$

Stress-like internal variable rate

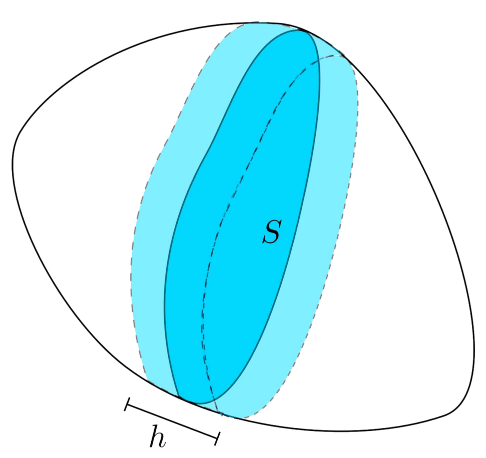
$$\dot{q}_\alpha = -H_\alpha(q_\alpha)\dot{r}_\alpha = -A_\alpha q_\alpha^\beta\dot{r}_\alpha$$

Energy dissipation given by a material parameter

$$W_{\mathrm{TOT}} = \int_\Omega \mathrm{d}\Omega\left(\int_{t=t_d}^{\infty}\mathcal{D}_{int}\mathrm{d}t\right) = \sum_{\alpha=g,f1,f2} SG_\alpha^f$$

Regularized softening modulus

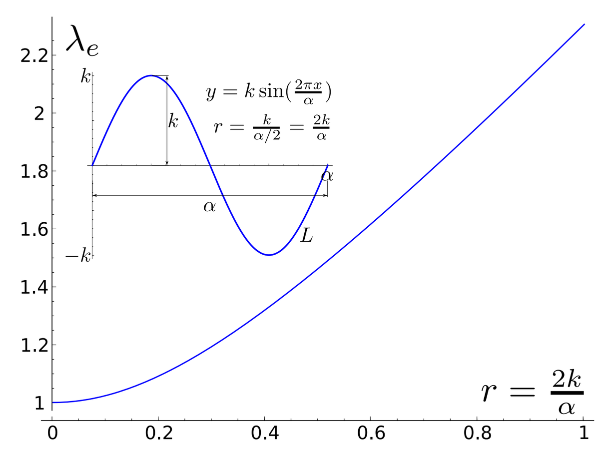
$$H_\alpha(q_\alpha(t)) = A_\alpha q_\alpha^\beta(t)h = -\frac{\left(q_\alpha^0\right)^{2-\beta}}{(2-\beta)}\frac{1}{G_\alpha^f}q_\alpha^\beta(t)h$$



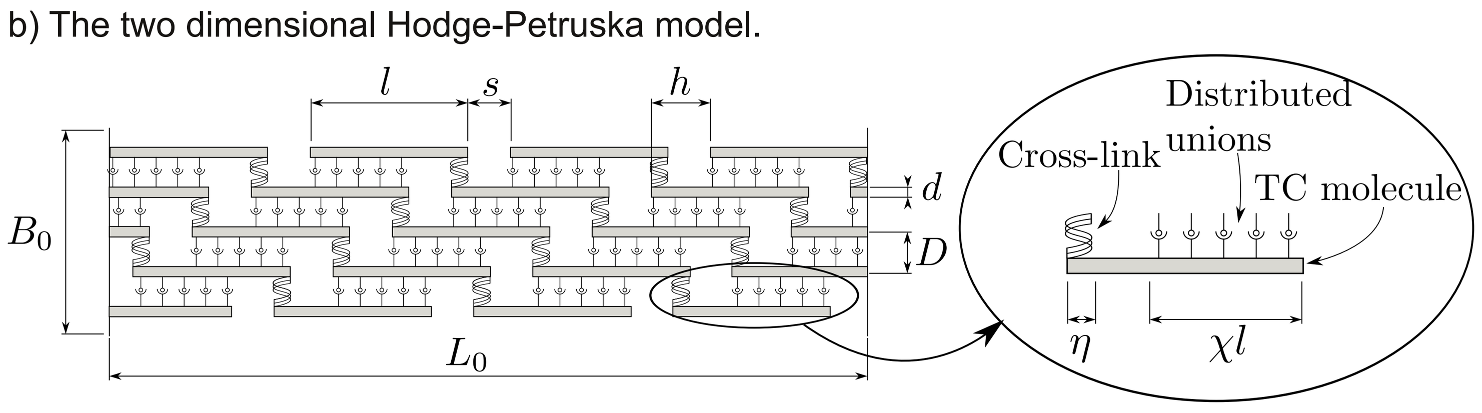
2.2 Mesoscopic characterization of inelastic behaviour

1. Fibril mesoscopic characterization

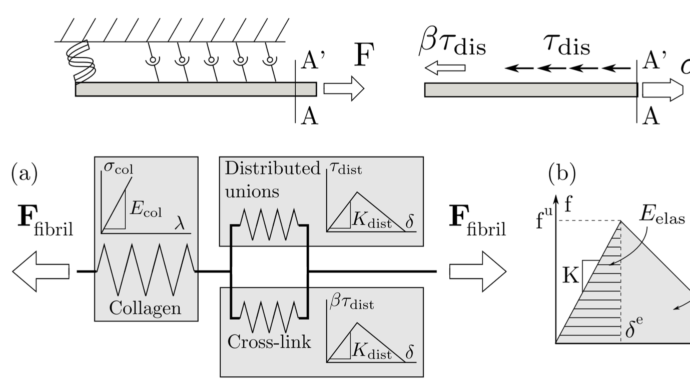
a) Elastic stretch threshold



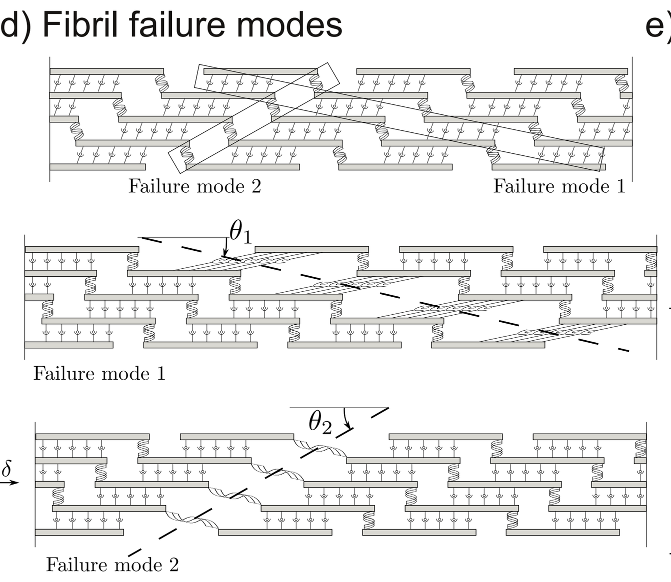
b) The two dimensional Hodge-Petruska model.



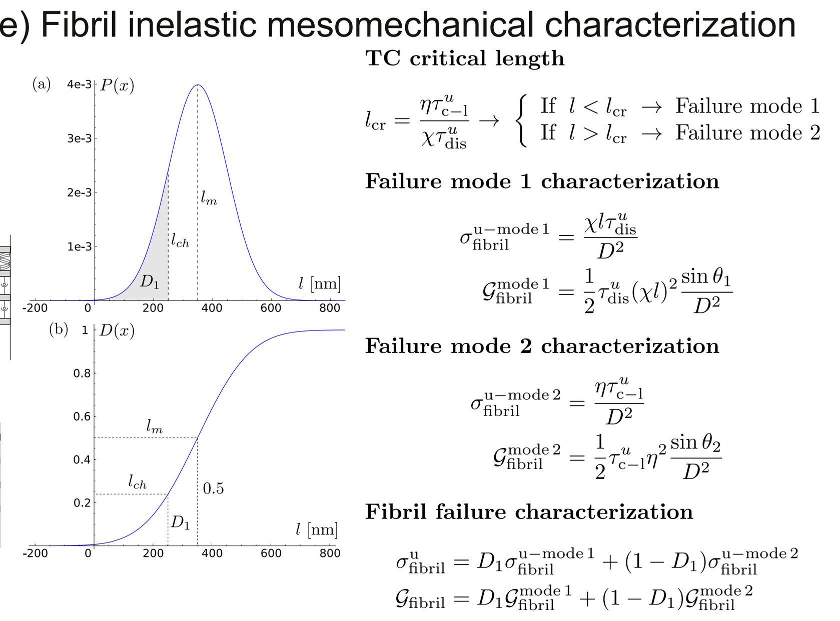
c) Fibril mesomechanical model



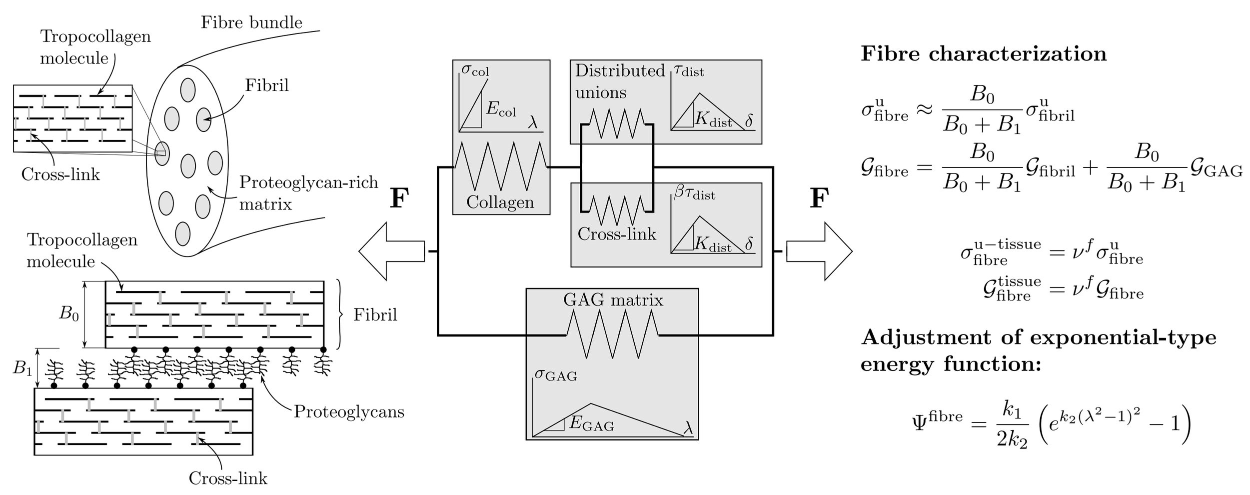
d) Fibril failure modes



e) Fibril inelastic mesomechanical characterization



2. Fibre and tissue characterization



3. Mesoscopic parameters

Cuadro 1: Geometrical properties of fibril components.

Description	Symbol	Value
Waviness parameter	r	0.33
Tropocollagen molecule length	l	300 nm
Tropocollagen molecule diameter	d	1,5 nm
Gap between tropocollagen molecules	s	40 nm
Distance of stagger	h	67 nm
Equilibrium distance between tropocollangen molecules	D	8 nm
Cross-link concentration length	η	6 nm

Cuadro 2: Micromechanical properties of fibril components.

Component	Elastic stiffness	Ultimate strength	Fracture energy (N·m)
Tropocollagen molecule	$E_{\mathrm{col}} = 50{,}0\ \mathrm{pN/Å^2}$ (a)	$\sigma^u = 10{,}0\ \mathrm{pN/Å^2}$ (b)	—
Distributed unions	$K_{\mathrm{dist}} = 1{,}18\ \mathrm{nN/Å}$ (c)	$\tau_{\mathrm{dist}}^u = 5{,}55\ \mathrm{pN/Å}$ (d)	$\mathcal{G}_{\mathrm{dist}} = \frac{1}{2}\tau_{\mathrm{dist}}^u(\chi l)^2$ (e)
Cross-links	$K_{\mathrm{dist}} = 1{,}18\ \mathrm{nN/Å}$ (c)	$\tau_{\mathrm{c-1}}^u = 69\ \mathrm{pN/Å}$ (f)	$\mathcal{G}_{\mathrm{c-1}} = \frac{1}{2}\tau_{\mathrm{c-1}}^u\eta^2$ (g)

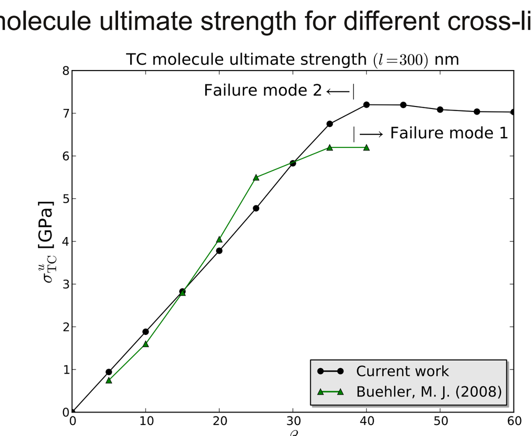
Cuadro 3: Properties of fibre components.

Component	Symbol	Value
Proteoglycan-rich matrix superfical density of fracture energy	$\mathcal{G}_{\mathrm{GAG}}$	10^3 pN/Å
Fibril width	B_0	180 nm
Proteoglycan-rich matrix width	B_1	100 nm
Fiber volume fraction of the tissue	ν^f	10^{-3}

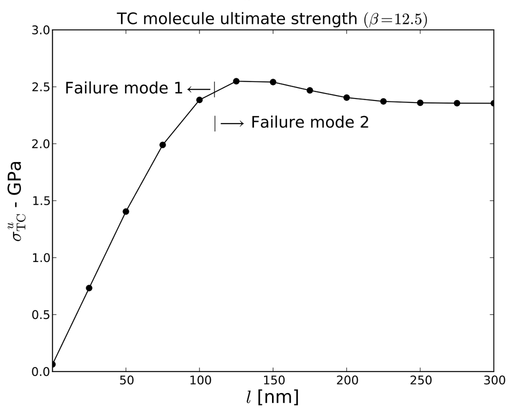
3. RESULTS

1. Fibril analysis

TC molecule ultimate strength for different cross-link densities

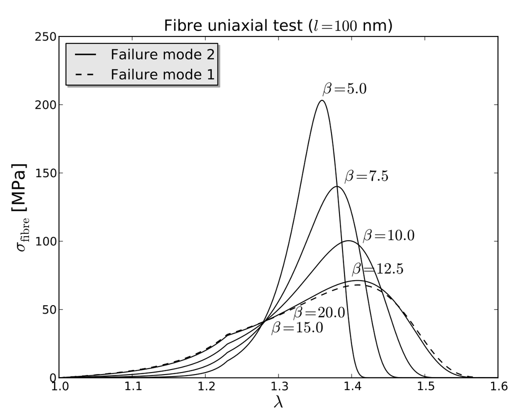


TC molecule ultimate strength for different molecular lengths

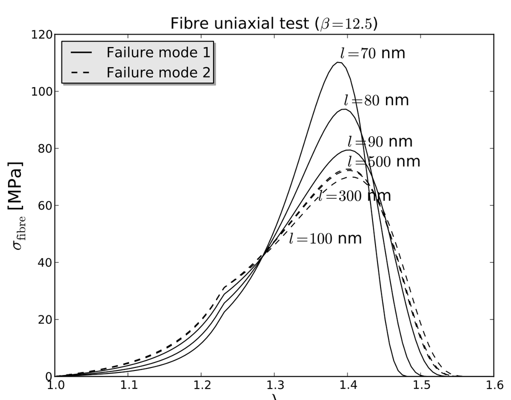


2. Fibre analysis

Fibre uniaxial test for different cross-link densities

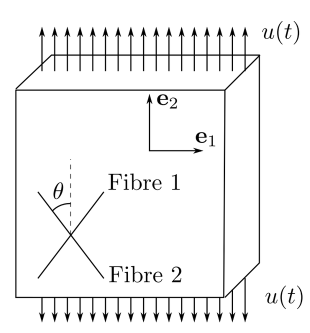


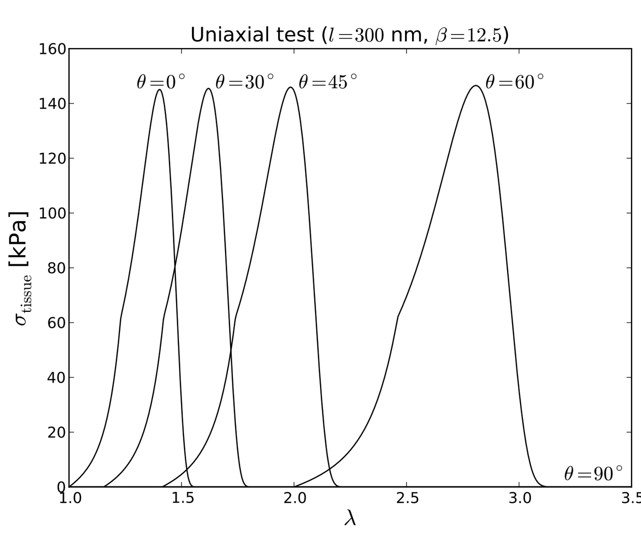
Fibre Uniaxial test for different molecular lengths



3. Tissue analysis

Uniaxial test for different fiber orientation angles





4. CONCLUSIONS

- Softening in damage models must be regularized in order to ensure the objectivity of the results.
- Energy dissipation in soft tissues should be considered as a needed material parameter and should be estimated.
- There is a dependence of the continuum response as a function of nanoscopic structural features.
- A hierarchical multi-scale approach is needed in order to define atomistically-informed continuum-scale mechanical properties of the soft tissues.